

## Collective motion in Ising-type Vicsek model

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### Abstract

This study shows collective phenomena of self-propelled particles using two-dimensional Ising-type Vicsek model, that we propose, by direct numerical simulation. Multiple particles can occupy one lattice point; self-propelled particles move one of eight directions in this model; particles change directions stochastically. They form a solitary wave at a high probability for aligning. Solitary waves suddenly change directions like a flip motion.

### 1. Introduction

Collectively moving animals and insects are fascinating. A simple model was proposed by Vicsek's group to study collective motion of self-propelled particles.<sup>[1]</sup> Agent-based models have been proposed and numerically studied to investigate dynamical behaviors of a large population of self-propelled particles.<sup>[2]</sup> Experimental study is also conducted, which is analyzed with Vicsek-like model.<sup>[3]</sup>

In Vicsek model, motion directions of particles align with neighborhood at high particle density. However, particles move randomly at low particle density. Then, directions of motion do not align for strong noise, which is for alignment direction. Therefore, order-disorder transition appears in this model.

Chaté's group found another state using Vicsek-like model with vector noise instead of scalar noise.<sup>[4]</sup> The state takes a band structure of dense particles on disorder state, which is like a solitary wave. We have shown that the solitary wave state appears in nonlinear Kramers equation; it is for the probability distribution of the position and velocity of self-propelled particles.<sup>[5]</sup> Furthermore, we have proposed a simple model of one-dimensional solitary wave state; the linear instability of the uniform disordered state leads to the formation of a solitary wave state.<sup>[6]</sup> Kuwayama's group has experimentally found biological soliton.<sup>[7]</sup>

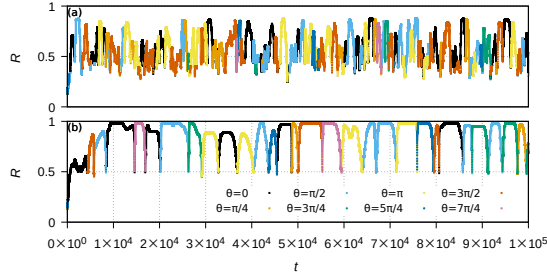
Experimental studies are expected to clarify the formation mechanism of a solitary wave.

Solon and Tailleur have proposed the active Ising model, in which self-propelled particles undergo a diffusion motion biased in the right or left direction and the updating is randomly performed like the Monte-Carlo simulation in the Ising model. Reversal phenomenon appear in the one-dimensional model.<sup>[8]</sup> The studies of the model are conducted,<sup>[9, 10]</sup> but the mechanism is unclear.

We have proposed Ising-type Vicsek model, which shows flip motions in one- and two-dimensional system.<sup>[11]</sup> Two-dimensional Ising-type Vicsek model is constrained by moving rule for simplicity, because particles move in only right or left directions. We show numerical result with generalized model and discuss the mechanism.

### 2. Ising-type Vicsek model

Ising model is the model describing phase transition of ferromagnet. One of plus-spin and minus-spin can occupy on a lattice point in the model. However, Ising-type Vicsek model takes multiple states, that is multiple particles can occupy one lattice point. The total number of particles is expressed as  $N$ . The lattice size is  $L_x$  and  $L_y$  in the  $x$  and  $y$  directions. Periodic boundary conditions are assumed. The number of particles at the  $x$ - $y$  coordinate  $(i, j)$  is expressed as  $n_{i,j}$ . The motion direction of a

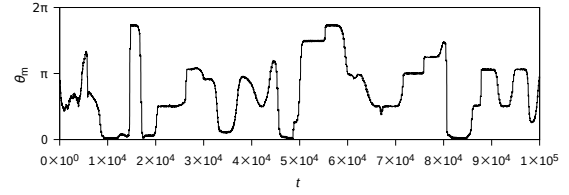


**Figure 1.** Time evolutions of alignment directions ratio of self-propelled particles at (a)  $g = 1$  and (b)  $g = 2$  for  $N = 100000$ ,  $L_x \times L_y = 200 \times 200$ . Colors show alignment directions. All particles move in the same direction at  $R = 1$ . The lower limit of  $R$  is  $1/8$ , because the motion direction of the particle takes one of the eight directions.

particle is expressed as  $\theta_k$ , where  $k$  is particle number. The particle number with moving direction  $\theta$  at  $(i, j)$  is expressed as  $n_{i,j,\theta}$  and the particle number with moving direction  $\theta$  at the neighboring sites around the  $(i, j)$  site is expressed as  $m_{i,j,\theta}$ , that is,

$$m_{i,j,\theta} = n_{i-1,j-1,\theta} + n_{i-1,j,\theta} + n_{i-1,j+1,\theta} + n_{i,j-1,\theta} + n_{i,j,\theta} + n_{i,j+1,\theta} + n_{i+1,j-1,\theta} + n_{i+1,j,\theta} + n_{i+1,j+1,\theta}. \quad (1)$$

The direction  $\theta_{max}$  taking the largest  $m_{i,j,\theta}$  is calculated at each site. A particle located at the  $(i, j)$  site moves at the next step to the direction  $\theta_{max}$  with probability  $p = e^g / (e^g + e^{-g})$ . Moreover, the particle can move to the direction  $\theta_{max} \pm \pi/4$  with probability  $(1-p)/3$ , to the direction  $\theta_{max} \pm \pi/2$  with probability  $(1-p)/6$ . For example, when  $\theta_{max}$  is 0, A particle located at the  $(i, j)$  site moves to x-axis forward ( $\theta = 0$ ) with probability  $p$ ; the particle can move to top-right direction ( $\theta = \pi/4$ ) or bottom-right direction ( $\theta = 7\pi/4$ ) with probability  $(1-p)/3$ , to y-axis forward direction ( $\theta = \pi/2$ ) or y-axis downward direction ( $\theta = 3\pi/2$ ) with probability  $(1-p)/6$ . That, is each particle moves toward the majority direction of



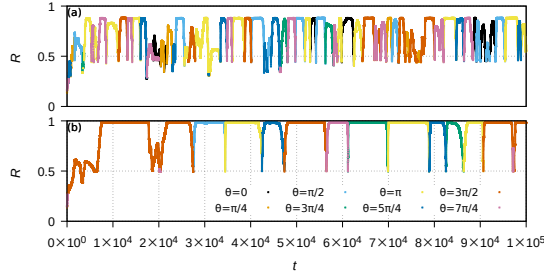
**Figure 2.** Time evolution of the mean motion direction of self-propelled particles at  $g = 2$ ,  $N = 100000$ ,  $L_x \times L_y = 200 \times 200$ . The change of  $\theta_m$  from  $7\pi/4$  to 0 means the direction changes by  $\pi/4$ .

neighboring particles with high probability  $p$ , however, the particle can change the direction with probability  $(1-p)$ . The parallel updating is performed in this model, that is all particle move at the same time. Our model is similar to the Vicsek model in that cooperative interaction and random forces are incorporated. The main difference is that the majority decision rule is adopted in our model.

Figure 1 shows that the moving particles form a cluster and the moving direction changes occasionally. The change of moving direction occurs suddenly. On the other hand, a steadily moving solitary-wave state is observed but the sequential flipping of moving direction is not reported in the original Vicsek model and the two-dimensional active Ising model. The alignment directions rate increases with increasing probability parameter  $g$ . The reversal time intervals increase with decreasing probability parameter  $g$ . The alignment directions ratio  $R$  is

$$R = \max_{\theta} \left( \frac{1}{N} \sum_{i,j} n_{i,j,\theta} \right), \quad (2)$$

where  $\max_{\theta}()$  is the normalized sum of  $n_{i,j,\theta}$  in  $\theta_{max}$ . Figure 2 shows the mean



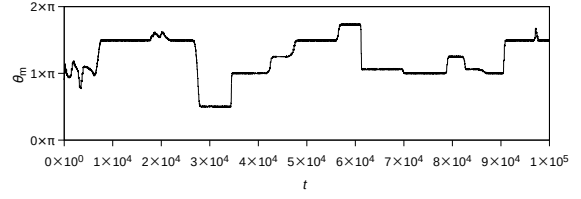
**Figure 3.** Time evolutions of alignment directions ratio of self-propelled particles with the next-nearest-neighbor couplings at (a)  $g = 1$  and (b)  $g = 2$  for  $N = 100000$ ,  $L_x \times L_y = 200 \times 200$ .

motion direction of all particle. The direction changes correspond with the changes in Fig. 1. The time evolution in the mean direction  $\theta_m$  seems to be discontinuous, but it is continuous.

We have shown the case of the nearest-neighbor interaction so far; however, similar flip motions of solitary waves are observed even in a model with both the nearest-neighbor and next-nearest-neighbor couplings. Hereafter, We show results of the model with the next-nearest-neighbor couplings. The interaction range is expanded. The particle number at the next-nearest-neighboring sites  $m'_{i,j,\theta}$  is

$$m'_{i,j,\theta} = m_{i,j,\theta} + n_{i-2,j-2,\theta} + n_{i-2,j-1,\theta} + n_{i-2,j,\theta} + n_{i-2,j+1,\theta} + n_{i-2,j+2,\theta} + n_{i-1,j-2,\theta} + n_{i-1,j+2,\theta} + n_{i,j-2,\theta} + n_{i,j+2,\theta} + n_{i+1,j-2,\theta} + n_{i+1,j+2,\theta} + n_{i+2,j-2,\theta} + n_{i+2,j-1,\theta} + n_{i+2,j,\theta} + n_{i+2,j+1,\theta} + n_{i+2,j+2,\theta}. \quad (3)$$

Figure 3 shows that the moving particles form a cluster more decidedly compared to Fig. 1. Figure 4 shows the frequency of flip motion is smaller compared to Fig. 2. The direction of motion of particles is easier to align with the next-nearest-neighbor couplings.



**Figure 4.** Time evolution of the mean motion direction of self-propelled particles with the next-nearest-neighbor couplings at  $g = 2$ ,  $N = 100000$ ,  $L_x \times L_y = 200 \times 200$ .

### 3. Summary

We have proposed Ising-type Vicsek model for two-dimensional self-propelled particles. We have found the sudden direction change of self-propelled particles in two-dimensional system. An inversely-moving solitary wave appears from the edge of the main solitary wave because of the collision between inverse moving wave with small amplitude and the edge of solitary wave. We have generalized Ising-type Vicsek model to reproduce the sudden cooperative change of moving direction.

### References

- [1] T. Vicsek, A. Czirók, E. Ben-Jacob, I. Cohen, and O. Shochet, Phys. Rev. Lett. **75**, 1226 (1995).
- [2] T. Vicsek and A. Zafeiris, Phys. Rep. **517**, 71 (2012).
- [3] Y. Sumino, K. H. Nagai, Y. Shitaka, D. Tanaka, K. Yoshikawa, H. Chaté, and K. Oiwa, Nature **483**, 448 (2012).
- [4] H. Chaté, F. Ginelli, G. Grégoire, F. Peruani, and F. Raynaud, Eur. Phys. J. B **64**, 451 (2008).
- [5] H. Sakaguchi and K. Ishibashi, J. Phys. Soc. Jpn. **86**, 114003 (2017).
- [6] H. Sakaguchi and K. Ishibashi, J. Phys. Soc. Jpn. **87**, 064001 (2018).
- [7] H. Kuwayama and S. Ishida, Sci Rep. **3** 2272 (2013).
- [8] A. P. Solon and J. Tailleur, Phys. Rev. Lett. **111**, 078101 (2013).
- [9] A. P. Solon and J. Tailleur, Phys. Rev. E **92**,

042119 (2015).

[10] A. P. Solon, H. Chaté, and J. Tailleur, Phys. Rev. Lett. **114**, 068101 (2015).

[11] H. Sakaguchi and K. Ishibashi, Phys. Rev. E

**100**, 052113 (2019).

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